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LETTER TO THE EDITOR

Interference in spin tunnelling of small magnetic particles

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Abstract. A previously studied (Enz and Schilling 1986 *J. Phys. C: Solid State Phys.* **19** 1765) magnetocrystalline anisotropic Hamiltonian with a magnetic field applied along the medium axis is reconsidered, with emphasis on the topological phase effect. A quantum inteference effect is revealed.

Tunnelling of magnetization in mesoscopic systems has attracted remarkable interest in recent years [1–3]. The total moment of a small ferromagnetic particle, for instance, may resonate between degenerate energy minima or out of a metastable direction, providing another example of the macroscopic quantum phenomena [4]. A notable subject in this area is whether the Berry phase, or Wess–Zumino, Chern–Simons term, gives rise to spin parity effects, like the well-known Haldane gap in quantum antiferromagnets. Loss *et al* [5] and von Delft and Henley [6] have confirmed this by demonstrating that, due to the interference between different paths, the tunnelling splitting can be quenched for magnetic particles with half-integer spin in the absence of external field. Later, Garg [7] studied the quenching of tunnelling when a magnetic field is applied along the hard axis of the particle and noted that the quenching need not be related to Kramers' degeneracy. Chudnovsky and DiVincenzo [8] discussed the situation when the external field is along the easy axis. In both cases the oscillation of the tunnelling splitting with the field is found. Garg [9] later considered the topological quenching more carefully and showed that the quenching in the tunnelling rate follows from a selection rule due to an underlying rotational symmetry. The spin parity effects and quantum propagation of Bloch walls in quasi-one-dimensional ferromagnets were studied by Braun and Loss [10] very recently, who found that the destructive interference between opposite chiralities suppresses nearest-neighbour hopping for half-integer spins. These studies have exhibited interesting quantum interference effects in small magnetic particles. In this note, we further consider the topological phase effect in the following Hamiltonian [11]:

$$H = -k_1 S_x^2 + k_2 S_z^2 - h S_y \qquad (1)$$

with positive k_1, k_2 and h, which may be interpreted as a magnetocrystalline anisotropic model with a magnetic field applied along the medium axis. Enz and Schilling [11, 12] have calculated the spin tunnelling rate of this problem, though a possible interference effect

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between different paths was not considered. For $0 \leqslant h \leqslant 2k_1 S$, the corresponding classical Hamiltonian of equation (1) possesses a two-fold-degenerate ground state at

$$\theta = \frac{\pi}{2},\ \phi_1 = \arcsin(h/2k_1 S) \qquad \text{and} \qquad \theta = \frac{\pi}{2},\ \phi_2 = \pi - \phi_1 \tag{2}$$

where S is the spin of the particle.

To start, we write the Euclidean tunnelling amplitude as a spin-coherent-state path integral:

$$K_E \equiv \langle \phi_2 | \mathrm{e}^{-HT} | \phi_1 \rangle = \int D\Omega \mathrm{e}^{-S_E} \tag{3}$$

where $S_E = \int_{-T/2}^{T/2} L \mathrm{d}\tau$ and

$$L = \mathrm{i}S\dot{\phi}(1 - \cos\theta) + E(\theta, \phi). \tag{4}$$

The first term of equation (4) defines the Wess–Zumino phase in the north-pole gauge.

In the semiclassical limit, the dominant contribution to the propagator comes from finite action solutions of the classical equation of motion (instantons). As noted by Enz and Schilling [11], since the configuration space is a circle, two types of instantons must be taken into account. We use A to denote the instanton passing through $\pi/2$ to ϕ_2 from ϕ_1, and B through $3\pi/2$ to ϕ_1 from ϕ_2. Correspondingly, we have two sorts of anti-instantons: A^- and B^-. The subtle point of the calculation is how to arrange the instantons and anti-instantons appropriately to satisfy the boundary condition when using the dilute instanton approximation [13]. We note that a finite action configuration starting from ϕ_1 and ending at ϕ_2 can be an arbitrary permutation of the pairs (AB), (AA^-), (B^-B) and (B^-A^-) plus an additional A or B^-. If we let $m,\ n,\ p$ and q be the numbers of A, B , A^- and B^- in a finite action configuration, and $i,\ j,\ k$ and l those of (AB), (AA^-), (B^-B) and (B^-A^-), we have

$$i + j = m \qquad i + k = n \qquad j + l = p \qquad k + l = q. \tag{5}$$

Therefore $m + q = n + p$ and only three variables are independent.

We may now write the expression for the tunnelling amplitude

$$\begin{aligned} K_E = \sqrt{\frac{\omega}{\pi\hbar}}\mathrm{e}^{-\omega_0 T/2} \sum_{mnpq} \frac{N(m,n,p,q)}{(m+n+p+q+1)!} &\big[(J_A k_A T \mathrm{e}^{-S_A})^{m+p+1}(J_B k_B T \mathrm{e}^{-S_B})^{n+q} \\ &\times \mathrm{e}^{\mathrm{i}S(\pi-2\phi_1)(m-p+1)}\mathrm{e}^{\mathrm{i}S(\pi+2\phi_1)(n-q)} + (J_A k_A T \mathrm{e}^{-S_A})^{m+p}(J_B k_B T \mathrm{e}^{-S_B})^{n+q+1} \\ &\times \mathrm{e}^{\mathrm{i}S(\pi-2\phi_1)(m-p)}\mathrm{e}^{\mathrm{i}S(\pi+2\phi_1)(n-q+1)}\big]\,\delta_{m+q,n+p} \end{aligned} \tag{6}$$

where ω_0 is the zero-point frequency in either well, J_A is the Faddeev–Popov determinant (or Jacobian) and S_A is the action of instanton A. The index B denotes corresponding quantities of instanton B. The definition of $k_A(k_B)$ is that of Coleman [13]. These quantities have been calculated by Enz and Schilling [11, 12] and they are not of interest here. To do the summation, we need $N(m,\ n,\ p,\ q)$ which represents the number of different finite action configurations for a given set of $\{m,\ n,\ p,\ q\}$. It can be calculated as

$$N(m,n,p,q) = \sum_{i=\max\{0,n-q\}}^{\min\{m,n\}} \frac{(i+j+k+l)!}{i!j!k!l!} \tag{7}$$

which has a simple result

$$N(m,n,p,q) = \frac{(m+q)!^2}{m!n!p!q!}. \tag{8}$$

Inserting equation (8) into (6), we obtain

$$K_E = \sqrt{\frac{\omega}{\pi\hbar}} \mathrm{e}^{-\omega_0 T/2} \sinh\left[J_A k_A T \mathrm{e}^{-S_A} + J_B k_B T \mathrm{e}^{-S_B} \cos(2S\pi)\right] \tag{9}$$

from which one can read off the tunnelling splitting

$$\Delta E = J_A k_A \mathrm{e}^{-S_A} + J_B k_B \mathrm{e}^{-S_B} \cos(2S\pi) \tag{10}$$

or

$$\Delta E = \Delta E_A + \Delta E_B \cos(2S\pi). \tag{10a}$$

This improves equation (8*a*) of Enz and Schilling [11]. Therefore the interference is either constructive or destructive depending only on the parity of the spin S. In the absence of the external field, ΔE_A equals ΔE_B, and for half-integer spin the quenching result [5, 6] is recovered naturally. Moreover, we observe no oscillation of ΔE with the external field, which is present when the field is along either the easy [7] or hard axis [8]. One expects a monotonic increase [12] of the tunnelling rate with the external field if it is not too large to destroy the barrier. For the topological effects in antiferromagnetic particles, as noted by Chudnovsky and DiVincenzo [8], the relevant quantity is the excess spin owing to the noncompensation of sublattices and the calculation here can apply to antiferromagnetic particles directly.

Various dissipative effects of the environment are important in the macroscopic quantum tunnelling of magnetism. The most important is, however, the interaction between spins of the particle and the spins of the environment (see [8] and references therein), since the change of a single 1/2 spin would transform the constructive interference to destructive or vice versa. Several studies [14, 15] suggest that the environmental spins suppress macroscopic quantum coherence severely. Whether the quantum interference effect can be observed experimentally is still an interesting problem deserving further study.

In summary, we have reconsidered the macroscopic quantum coherence in a magnetocrystalline anisotropic Hamiltonian with an external magnetic field applied along the medium axis, which has been studied by Enz and Schilling. The quantum interference between topologically different paths is revealed.

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